

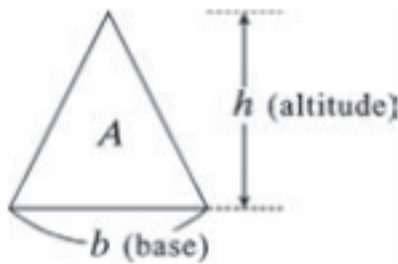
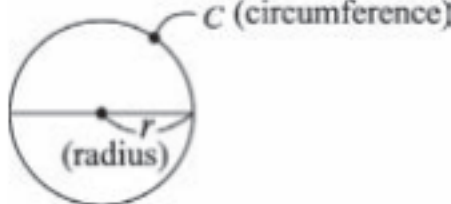
LEVEL G

Topic	Formulas & Notes	Reference
Addition and Subtraction of Positive and Negative Numbers 1	$+5 - 2 = +3, \quad +2 - 5 = -3$ $-5 + 2 = -3, \quad -2 + 5 = +3$ ‘+’ or ‘-’ in front of a number is the sign of the number. ‘+’ is read as ‘plus’, ‘-’ is read as ‘minus’, and ‘=’ is read as ‘equals’. $0 - 5 = -5, \quad -2 - 5 = -7$	G21 G22a G24a
Addition and Subtraction of Positive and Negative Numbers 2	Ex. $\frac{1}{5} - 2\frac{4}{5} = -2\frac{3}{5}$ $\frac{4}{5} - 2\frac{1}{5} = \frac{4}{5} - 1\frac{6}{5} = -1\frac{2}{5}$	G31a
Addition and Subtraction of Positive and Negative Numbers 3	When removing parentheses, remember to change the sign of the number within the parentheses if there is a negative sign in front of the parentheses. $+(-2) - (+4) = -2 - 4 = -6$ $-(-2) + (+4) = 2 + 4 = 6$	G46b
Multiplication / Division of Positive and Negative Numbers	For multiplication and division excluding 0, if there is: • an odd number of ‘-’ signs (1, 3, 5, ...), the answer is ‘-’ • an even number of ‘-’ signs (2, 4, 6, ...), the answer is ‘+’	G63b G72b
Values of Algebraic Expressions	Replacing letters with numbers is called <i>substitution</i>. The result of the calculation is called the <i>value</i> of the expression. Ex. When $a = 3$ and $b = 2$, $6a - 4b = 6 \times 3 - 4 \times 2 = 18 - 8 = 10$	G101a G118a
<i>Algebraic Expressions</i>	(1) $2 \times a = 2a, \quad x \times y \times z = xyz$ (2) $1 \div a = \frac{1}{a}, \quad b \div a = \frac{b}{a}$ (3) $a \times a = a^2, \quad a \times a \times b = a^2b$	
Simplifying Algebraic Expressions 1	Terms with the same variable can be combined as follows: Ex. $x + x + x = 3x, \quad 3a + 2a = 5a, \quad 3a - 5a = -2a$	G121a
<i>Algebraic Notation</i>	(1) $1x$ and $1a$ are written as x and a $-1x$ and $-1a$ are written as $-x$ and $-a$ (2) When simplifying algebraic expressions containing fractions, use improper fractions rather than mixed numbers. $\frac{7}{3}a$ is not changed to $2\frac{1}{3}a$	G121 G128a

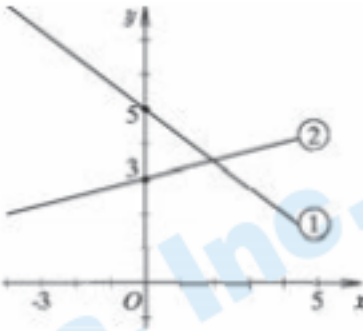
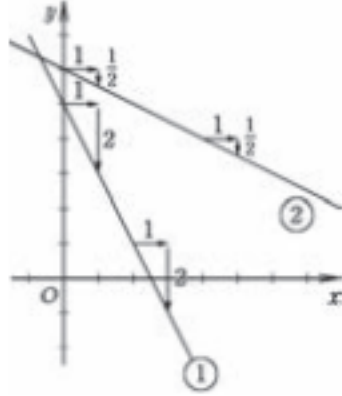
LEVEL G

Topic	Formulas & Notes	Reference
Simplifying Algebraic Expressions 2	Write answers in alphabetical order, $(a, b, c$ or $x, y, z)$ and in order of decreasing degree of the specific variable (x^2, x) .	G135a
Simplifying Algebraic Expressions 3	Expressions such as $a(b + c)$ and $(b + c)a$ can be simplified as follows: $a(b + c) = ab + ac$ $(b + c)a = ab + ac$	G143a
Linear Equations 1	$2x = 6$ [Sol] $x = 6 \times \frac{1}{2}$ $x = 3$ To change the equation into the form “$x =$ a number”, we transform $2x$ into x. We do this by multiplying both sides of the equation by $\frac{1}{2}$.	G161b
	$2x + 8 = 14$ $\xrightarrow{\quad \quad \quad}$ $2x = 14 - 8$ The sign has to be changed when a term is transferred to the other side of the ‘$=$’ sign. This is called <i>transposition</i>.	G163a
Linear Equations 3	To remove denominators, multiply each side of the equation by the <i>LCM</i> . Ex. $\frac{1}{2}x + \frac{1}{4} = \frac{1}{6}x + 1$ [Sol] Multiply each side by 12.  The LCM of 2, 4 and 6 is 12. $\left(\frac{1}{2}x + \frac{1}{4}\right) \times 12 = \left(\frac{1}{6}x + 1\right) \times 12$ $6x + 3 = 2x + 12$ $6x - 2x = 12 - 3$ $4x = 9$ $x = \frac{9}{4}$	G181a
<i>Linear Equations</i>	In order for a given equation to remain a true statement, if an operation is performed on one side of the equation, the exact operation must also be performed on the other side of the equation. Given the equation $A = B$, (1) $A + m = B + m$ (2) $A - m = B - m$ (3) $Am = Bm$ (4) $\frac{A}{m} = \frac{B}{m} (m \neq 0)$	

LEVEL H

Topic	Formulas & Notes	Reference
Transforming Equations 2	Area of a triangle: $A = \frac{1}{2}bh$ 	H36a
	Circumference of a circle: $C = 2\pi r$ (where $\pi \approx 3.14$) 	H36b
Simultaneous Linear Equations in Two Variables 1	$\begin{cases} 5x + 2y = 11 & \dots \textcircled{1} \\ 3x + 2y = 5 & \dots \textcircled{2} \end{cases}$ [Sol] $\textcircled{1} - \textcircled{2}: 2x = 6$ $x = 3$ Substituting this into $\textcircled{1}$, $5 \times 3 + 2y = 11$ $2y = -4$ $y = -2$ Ans. $(x, y) = (3, -2)$ Procedure: 1. Number each equation. 2. Subtract one equation from the other to remove one variable. 3. Solve for x. 4. Substitute the value of x in one of the given equations and solve for y. 5. Write the answer.	H41a
Simultaneous Linear Equations in Two Variables 5	$\begin{cases} y = 3x - 1 & \dots \textcircled{1} \\ 5x - y = 5 & \dots \textcircled{2} \end{cases}$ [Sol] Substituting $\textcircled{1}$ into $\textcircled{2}$, $5x - (3x - 1) = 5$ $2x = 4$ $x = 2$ Substituting this into $\textcircled{1}$, $y = 3 \times 2 - 1 = 5$ Ans. $(x, y) = (2, 5)$	H81a
Inequalities 1	Given $A < B$, 1) $A + C < B + C$ or $A - C < B - C$. You can add the same number to or subtract the same number from both sides. 2) When $C > 0$, $AC < BC$ or $\frac{A}{C} < \frac{B}{C}$ You can multiply or divide both sides by the same number. 3) When $C < 0$, $AC > BC$ or $\frac{A}{C} > \frac{B}{C}$ When multiplying or dividing both sides by a negative number, you need to reverse the inequality symbol.	H121 H122 H123

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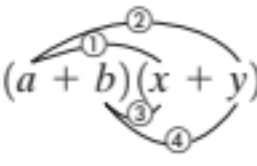
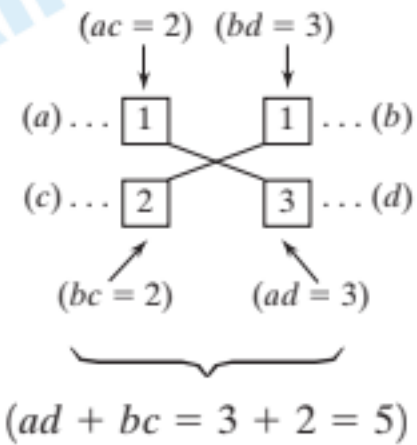
Topic	Formulas & Notes	Reference
Functions and Graphs 1	If the relation between x and y can be expressed as $y = mx$, the relation is called a <i>direct variation</i> and m is the <i>constant of variation</i> . The graph of a direct variation, $y = mx$, is a line that passes through the origin.	H147a
	If the relation between x and y can be expressed as $y = \frac{k}{x}$, the relation is called an <i>inverse variation</i> and k is the <i>constant of variation</i> .	H149a
Functions and Graphs 2	The <i>y-intercept</i> of a line is the y-coordinate of the point where the line intersects the y-axis. In line ①, the <i>y-intercept</i> is 5. In line ②, the <i>y-intercept</i> is 3. 	H151a
	The slope of a line is expressed by the ratio $\frac{\text{rise}}{\text{run}}$ (the steepness of the line). In line ①, the slope is 3; the line rises 3 units for every unit that it runs horizontally. In line ②, the slope is $\frac{1}{2}$; the line rises $\frac{1}{2}$ unit for every unit that it runs horizontally. 	H152a
	Lines ① and ② have negative slopes. The slope of line ① is -2 because the line falls 2 units for every unit that it runs horizontally. The slope of line ② is $-\frac{1}{2}$ because the line falls $\frac{1}{2}$ unit for every unit that it runs horizontally. 	H153a
	■ If the relation between x and y is expressed as $y = mx + b$, the relation is called a linear function. ■ In the above equation, m is the <i>slope</i>, and b is the <i>y-intercept</i>. ■ $y = mx + b$ is the equation of a line that passes through point $(0, b)$.	H156a

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Topic	Formulas & Notes	Reference
Functions and Graphs 3	The slope of a line that passes through the two points (a, b) and (c, d) can be found as follows: $\text{Slope} = \frac{d - b}{c - a}$	H165b
Functions and Graphs 4	An equation $ax + by + c = 0$ can be transformed to: $y = -\frac{a}{b}x - \frac{c}{b}$ The slope of the line of the above equation is $-\frac{a}{b}$ and the y-intercept is $-\frac{c}{b}$.	H171b
	Given an equation $ax + by + c = 0$, 1. If $a = 0$, the above equation is transformed to $y = -\frac{c}{b}$, its line passes through $\left(0, -\frac{c}{b}\right)$ and is parallel to the x-axis. The slope is 0. 2. If $b = 0$, the above equation is transformed to $x = -\frac{c}{a}$, its line passes through $\left(-\frac{c}{a}, 0\right)$ and is parallel to the y-axis. The slope is <i>undefined</i>.	
	The solution of simultaneous equations represents the coordinates of the point of intersection of the lines of the simultaneous equations.	H172b
	If the slopes of lines are equal, the lines are <i>parallel</i>. Line $y = mx + b$ is parallel to line $y = mx$.	H178a
Simplifying Monomials and Polynomials 1	Ex. $3a^2b \times 4a^3 = 12a^5b$ Procedure for simplifying: 1) Determine the sign of the answer. 2) Calculate the numbers in front of the letters. 3) Calculate the exponents.	H183a

[NOTES]

LEVEL I

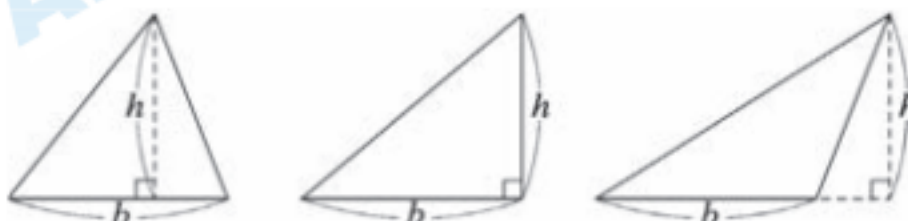
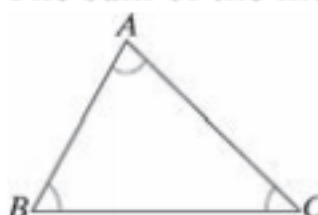
Topic	Formulas & Notes	Reference
Multiplication of Polynomials	 $(a + b)(x + y) = \frac{ax}{\textcircled{1}} + \frac{ay}{\textcircled{2}} + \frac{bx}{\textcircled{3}} + \frac{by}{\textcircled{4}}$	I11a
Multiplication using Formulas	$(a + b)^2 = a^2 + 2ab + b^2$	I21a
	$(a - b)^2 = a^2 - 2ab + b^2$	I22a
	$(a + b)(a - b) = a^2 - b^2$	I26a
	$(x + a)(x + b) = x^2 + (a + b)x + ab$	I27a
	$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$	I29a
Factorization 1	$ax + ay = a(x + y)$	I31a
	$(a + b)x + (a + b)y = (a + b)(x + y)$	I33a
	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	I36a
Factorization 2 <i>Difference of Two Squares</i>	$a^2 - b^2 = (a + b)(a - b)$	I41a
	$x^2 + (a + b)x + ab = (x + a)(x + b)$	I44a
Factorization 3 <i>Cross Multiplication Method</i>	$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$ Ex. $2x^2 + 5x + 3 = (x + 1)(2x + 3)$  This method can be used to find the coefficients $a, b, c,$ and d. We estimate the coefficients so that $ac = 2$, $bd = 3$, and $ad + bc = 5$.	I51a
Factorization 5	$y - x = -x + y = -(x - y)$	I71b
	$ \begin{aligned} &x^2(a - b) + y^2(b - a) \\ &= x^2(a - b) - y^2(a - b) \\ &= (a - b)(x^2 - y^2) \\ &= (a - b)(x + y)(x - y) \end{aligned} $  To find the common factor, try changing the signs and order of the terms.	I72a
	$ax + ay + bx + by = a(x + y) + b(x + y)$ $= (x + y)(a + b)$	I76a

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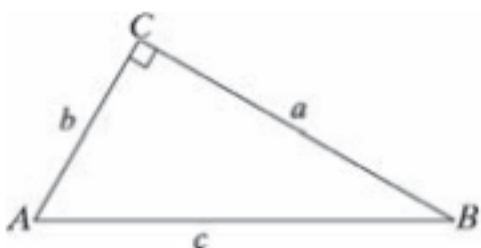
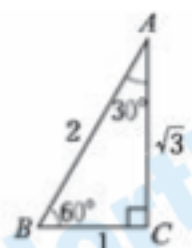
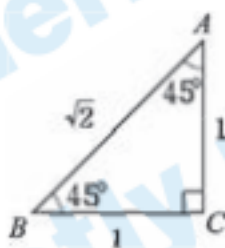
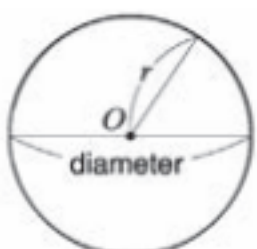
Topic	Formulas & Notes	Reference
Square Roots 1	A <i>square root</i> of a number is a number that when squared is equal to the given number. Ex. The square roots of 25 are 5 and -5	I81a
	$\sqrt{a}$ represents the <i>positive square root</i> of a number a. (The negative square root is represented by $-\sqrt{a}$.) Ex. $\sqrt{144} = 12$	I82a
Square Roots 2 <i>Multiplication of Square Roots</i>	For any positive numbers a and b , $\sqrt{a}\sqrt{b} = \sqrt{ab}$.	I91a
<i>Prime Factorization</i>	1) A <i>prime number</i> is an integer greater than 1 that has no positive integral factors other than itself and 1. Ex.(2, 3, 5, 7, 11, 13, 17...) 2) The factorization of an integer over its positive prime numbers is called <i>prime factorization</i>. Ex. $45 = 9 \times 5 = 3 \times 3 \times 5 = 3^2 \times 5$	I94a
<i>Addition and Subtraction of Square Roots</i>	$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ $\sqrt{18} + \sqrt{50} = 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$ $\sqrt{12} - \sqrt{48} = 2\sqrt{3} - 4\sqrt{3} = -2\sqrt{3}$	I99a
Square Roots 3 <i>Division of Square Roots</i>	For any <u>positive</u> numbers a and b , $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	I106a
<i>Rationalizing the Denominator</i>	$\frac{b}{\sqrt{a}} = \frac{b\sqrt{a}}{\sqrt{a}\sqrt{a}} = \frac{b\sqrt{a}}{a} \quad (a > 0)$ Multiply both the denominator and the numerator by the radical in the denominator, $\sqrt{a}$.	I107a
Quadratic Equations 1	Given a quadratic equation of the form $ax^2 + bx + c = 0 \quad (a \neq 0),$ If the left side of the equation can be factored as $(dx + e)(fx + g) = 0$, then the solution of the equation is: $x = -\frac{e}{d}, \quad x = -\frac{g}{f}$	I111a

[NOTES]

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Topic	Formulas & Notes	Reference
Quadratic Equations 2 <i>Completing the Square</i>	$2x^2 - 3x - 1 = 0$ [Sol] $x^2 - \frac{3}{2}x = \frac{1}{2}$ $\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} = \frac{1}{2}$ $\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$ $\vdots$ (continue solving as usual) Transpose the numerical (constant) term to the right side of the equation, and divide both sides of the equation by the coefficient of the square term, in this case 2. Add a constant term to the left side so that it includes the square of a polynomial. To keep the equation a true statement, subtract the term from the same side. (Take the coefficient of the x -term, divide it by 2, and square the result.) $\left(-\frac{3}{2}\right) \cdot \frac{1}{2} = -\frac{3}{4}, \left(-\frac{3}{4}\right)^2 = \frac{9}{16}$	I124a I129a
Quadratic Equations 3 <i>Quadratic Formula</i>	Given $ax^2 + bx + c = 0$ ($a \neq 0$), $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	I131a
Graphs of Quadratic Functions	1) The graph of $y = ax^2$ ($a \neq 0$): Vertex: $(0, 0)$, axis of symmetry: line $x = 0$ (the y -axis) 2) The graph of $y = a(x - p)^2 + q$ ($a \neq 0$): Vertex: (p, q) , axis of symmetry: line $x = p$ 3) The graph of $y = ax^2 + bx + c$ ($a \neq 0$) $= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$ Vertex: $\left(-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a}\right)$, axis of symmetry: line $x = -\frac{b}{2a}$	I150 I153b
The Pythagorean Theorem 1 <i>Areas of Triangles</i>	$A = \frac{1}{2}bh$ [where b is the base and h is the altitude (or height)] 	I171a
<i>The Triangle-Sum Theorem</i>	The sum of the measures of the angles of a triangle is 180° .  $m\angle A + m\angle B + m\angle C = 180^\circ$	I171b

LEVEL I

Topic	Formulas & Notes	Reference
<i>Pythagorean Theorem</i>	In a <i>right triangle</i>, the square of the length of the <i>hypotenuse</i> (c) equals the sum of the squares of the other two sides (a and b) . If $m\angle C = 90^\circ$, $c^2 = a^2 + b^2$ 	I172a
<i>Ratios and Proportions</i>	The expression $a : b$ represents the <i>ratio</i> of a to b. $a : b = \frac{a}{b} \quad (b \neq 0)$ If $a : b = c : d$, then $ad = bc$ (Note: $a : b = c : d$ is called a <i>proportion</i> and can be written as $\frac{a}{b} = \frac{c}{d}$ ($b \neq 0, d \neq 0$))	I179a I179b
<i>Special Right Triangles</i>	30°-60°-90°  $1 : \sqrt{3} : 2$ Ratio of the length of each side to the other 45°-45°-90°  $1 : 1 : \sqrt{2}$ Ratio of the length of each side to the other (isosceles right triangle)	I180a
The Pythagorean Theorem 2 <i>Circumference and Area of Circles</i>	For a circle with radius r, ($\pi \approx 3.14$) Circumference, $C = 2\pi r$ Area, $A = \pi r^2$ 	I189a
<i>Area of a Sector</i>	For a circle with radius r, the area, A, of a sector whose central angle measures m° is given by: $A = \frac{m}{360} \pi r^2$ 	I189b
The Pythagorean Theorem 3 <i>Distance</i>	The distance (AB) between points $A(a, b)$ and $B(c, d)$: $AB = \sqrt{(c - a)^2 + (d - b)^2}$	I191a

LEVEL J

Topic	Formulas & Notes	Reference								
Expansion of Polynomial Products <i>Laws of Exponents</i>	For any number a and b , when m and n are positive integers: $a^m \times a^n = a^{m+n}, \quad (a^m)^n = a^{m \times n}, \quad (ab)^m = a^m b^m$	J1a								
<i>Distributive Property</i>	For any numbers a, b , and c : $c(a + b) = ca + cb$									
	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$	J4b								
	$(a + b)(a - b) = a^2 - b^2$	J5a								
	$(x + a)(x + b) = x^2 + (a + b)x + ab$	J6a								
	$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	J7a J8a								
	$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	J7b J8a								
	$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$	J9b								
Factorization 1	$x^2 + (a + b)x + ab = (x + a)(x + b)$	J11a								
	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	J15a								
<i>Difference of Two Squares</i>	$a^2 - b^2 = (a + b)(a - b)$	J17a								
<i>Treating Multiple Terms as a Single Quantity</i>	<table><tr><th>Given</th><th>Terms to be treated as single quantities</th></tr><tr><td>$(x + y)^2 - 3(x + y) - 10$</td><td>$(x + y)$</td></tr><tr><td>$(a + b)^2 - (c + d)^2$</td><td>$(a + b)$ and $(c + d)$</td></tr><tr><td>$x^4 - y^4 = (x^2)^2 - (y^2)^2$</td><td>$x^2$ and y^2</td></tr></table>	Given	Terms to be treated as single quantities	$(x + y)^2 - 3(x + y) - 10$	$(x + y)$	$(a + b)^2 - (c + d)^2$	$(a + b)$ and $(c + d)$	$x^4 - y^4 = (x^2)^2 - (y^2)^2$	x^2 and y^2	J11b J18a J19a
Given	Terms to be treated as single quantities									
$(x + y)^2 - 3(x + y) - 10$	$(x + y)$									
$(a + b)^2 - (c + d)^2$	$(a + b)$ and $(c + d)$									
$x^4 - y^4 = (x^2)^2 - (y^2)^2$	x^2 and y^2									
Factorization 2 <i>Factoring out ‘−1’</i>	$(b - a) = -(a - b)$ $(b - a)^2 = (a - b)^2$ $(b - a)^3 = -(a - b)^3$	J22b J23a J25a								
<i>Factorization by Grouping</i>	$ax + ay + bx + by = a(x + y) + b(x + y) = (x + y)(a + b)$, or $ax + ay + bx + by = x(a + b) + y(a + b) = (a + b)(x + y)$ $\overset{\wedge}{x^3} + \overset{\wedge}{4x^2} + \overset{*}{2x} + \overset{*}{8} = x^2(x + 4) + 2(x + 4) = (x + 4)(x^2 + 2)$ (Note: here the symbols $\wedge$ and $*$ are used to mark the terms that are grouped together in the intermediate step of this example.)	J26a J28a								
Factorization 3 <i>Arranging in Standard Polynomial Form</i>	Given $x^2 + 5xy + 6x + 6y^2 + 13y + 5$, With x as the variable: $x^2 + (5y + 6)x + (6y^2 + 13y + 5)$ With y as the variable: $6y^2 + (5x + 13)y + (x^2 + 6x + 5)$	J34a								

LEVEL J

Topic	Formulas & Notes	Reference
Factorization 4 <i>Sum and Difference of Two Cubes</i>	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	J47a J48a
Factorization 5 <i>Factoring Fourth Degree Polynomials</i>	$x^4 - 6x^2 + 1$ $= x^4 - 2x^2 + 1 - 4x^2$ $= (x^2 - 1)^2 - (2x)^2$ $\vdots$ (continue factoring as usual) Break up the x^2 term into two terms forming a binomial square minus a monomial square; i.e., the difference of two squares.	J51a
<i>Treating Multiple Terms as a Single Quantity</i>	$(x^2 + x - 6)(x^2 + x - 2) + 3 \longleftarrow \text{Let } x^2 + x = A$ $= (A - 6)(A - 2) + 3$ $= A^2 - 8A + 15 = (A - 3)(A - 5) = (x^2 + x - 3)(x^2 + x - 5)$	J53a
Fractional Expressions	$\frac{mA}{mB} = \frac{A}{B}$	J61a
	$\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}, \quad \frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C} = \frac{A \times D}{B \times C}$	J64a
	$\frac{A}{C} \pm \frac{B}{C} = \frac{A \pm B}{C}$	J66a
	$\frac{1}{a} + \frac{1}{b} = \frac{b}{ab} + \frac{a}{ab} = \frac{a + b}{ab}$	J66b
Irrational Numbers 1 <i>Rationalizing the Denominator</i>	Ex. $\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{15}}{5}$	J74a
	$\frac{c}{\sqrt{a} \pm \sqrt{b}} = \frac{c(\sqrt{a} \mp \sqrt{b})}{(\sqrt{a} \pm \sqrt{b})(\sqrt{a} \mp \sqrt{b})} = \frac{c(\sqrt{a} \mp \sqrt{b})}{a - b} \quad (a > 0, b > 0)$	J75a
<i>Double Radicals</i>	$\sqrt{7 + 2\sqrt{10}} = \sqrt{5} + \sqrt{2}$ $\begin{array}{cc} \uparrow & \uparrow \\ 5 + 2 & 5 \times 2 \end{array}$ (The two resulting radicands must add to give 7, and multiply to give 10.)	J77b
Irrational Numbers 2	$\sqrt{a^2} = \begin{cases} a & (\text{when } a \geq 0) \\ -a & (\text{when } a < 0) \end{cases} \quad \text{Note: } [\sqrt{0} = 0]$	J86b
	$\sqrt{(a - b)^2} = \begin{cases} a - b & (\text{when } a \geq b) \\ -(a - b) & (\text{when } a < b) \end{cases}$	J87b
Quadratic Equations 1 <i>Quadratic Formula I</i>	When $ax^2 + bx + c = 0$ ($a \neq 0$), $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	J97b

LEVEL J

Topic	Formulas & Notes	Reference
Quadratic Equations 2 <i>Quadratic Formula II</i>	When $ax^2 + 2b'x + c = 0$ ($a \neq 0$), $x = \frac{-b' \pm \sqrt{b'^2 - ac}}{a}$ Use <i>Quadratic Formula II</i> to solve quadratic equations when the coefficient of x is an even number.	J102a
Quadratic Equations and Complex Numbers <i>Imaginary Number</i>	Positive and negative numbers, and zero, belong to the set of <i>real numbers</i> . All positive and negative numbers become positive when squared. 0 becomes 0 when squared. The <i>imaginary number</i> , i , is not a real number, and becomes negative when squared. $i^2 = -1$ ($\sqrt{-1} = i$)	J111a
	When $a \geq 0$, $\sqrt{-a} = \sqrt{a}i$	J111b
	$\sqrt{a}\sqrt{b} = \sqrt{ab}$ (unless $a < 0$ and $b < 0$)	J112b
	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ (unless $a > 0$ and $b < 0$)	J113a
<i>Complex Numbers</i>	$3 + 2i$ and $4 - 3i$ are <i>complex numbers</i> . $a + bi$ consists of real and imaginary parts; a and b are real parts. $a + bi$ is a real number when $b = 0$ (since $a + 0 \cdot i = a$) When $b \neq 0$, $a + bi$ is a complex number.	J113b
	$a + bi = c + di \Leftrightarrow a = c, b = d$ (Where a, b, c, d $a + bi = 0 \Leftrightarrow a = 0, b = 0$ are real numbers.)	J115b
	If a quadratic equation in the form $ax^2 + bx + c = 0$ has complex roots, and one of them is $x = \alpha + \beta i$, then the other one is $x = \alpha - \beta i$.	J119b
Discriminant	Given $ax^2 + bx + c = 0$, $D = b^2 - 4ac$ When $D > 0$, there are two different real number solutions. When $D = 0$, there is one repeated solution. When $D < 0$, there are two different complex number solutions.	J121b
	Given $ax^2 + 2b'x + c = 0$, $D' = \frac{D}{4} = b'^2 - ac$	J123a
Root-Coefficient Relationship	Given $ax^2 + bx + c = 0$ ($a \neq 0$), assuming the roots are α and β : $\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$	J131b
Dividing Polynomials	In the division of $(x^3 - x^2 - 1) \div (x - 2)$ the quotient is $x^2 + x + 2$ with remainder 3. This relationship may be written as: $x^3 - x^2 - 1 = (x - 2)(x^2 + x + 2) + 3$	J154a

LEVEL J

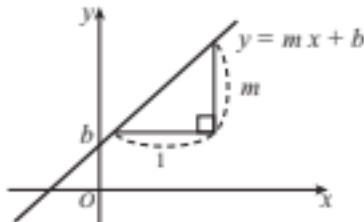
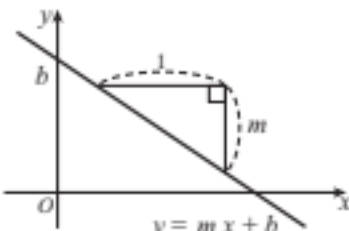
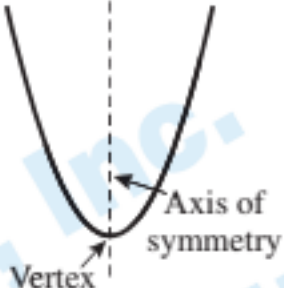
Topic	Formulas & Notes	Reference
Remainder Theorem	If a polynomial $P(x)$ is divided by a term $(x - a)$, we can express the relation with the quotient $Q(x)$, and the remainder R as: $P(x) = (x - a)Q(x) + R$ When $x = a$, $P(a) = (a - a)Q(x) + R = R$. Therefore,	J162b
<i>Remainder Theorem</i>	When polynomial $P(x)$ is divided by $(x - a)$, the remainder is $P(a)$.	J163a
	The <i>Remainder Theorem</i> can only be used when dividing by a linear expression. When dividing by a quadratic expression, use long division.	J164b
<i>General Form of a Remainder</i>	-When the divisor is a second-degree expression (a quadratic) the remainder must be of degree one or less, in the form: $ax + b$ -When the divisor is a third-degree expression (a cubic) the remainder must be of degree two or less, in the form: $ax^2 + bx + c$	J165b
Factor Theorem	When $P(a) = 0$, it means that the polynomial $P(x)$ has a factor, $(x - a)$.	J172a
<i>Factor Theorem</i>	Therefore, a polynomial, $P(x)$, has a factor $(x - a)$, if and only if, $\Leftrightarrow P(a) = 0$.	
	Given a polynomial $P(x)$, to determine a value, a, at which $P(a) = 0$, we first check the coefficient of the highest power of the variable in the polynomial. ► If it is 1, we try a number that is a factor of the constant in the polynomial. ► If it is greater than 1, we try a number such that: - Its numerator is a factor of the constant in the polynomial. And, - Its denominator is a factor of the coefficient of the highest power of the variable in the polynomial.	J172a J174a
Proof of Identities and Equalities	There are 2 ways to determine the value of the coefficients to show that an equality is an identity:	J181a
<i>Coefficient Comparison Method</i>	① Given $(ax + b)(x + 1) = 3x^2 + 5x + 2$, $ax^2 + (a + b)x + b = 3x^2 + 5x + 2$ Matching the coefficients of x^2, x, and the constants of the LHS and RHS: $a = 3$, $a + b = 5$, $b = 2$ (Ans. $a = 3$, $b = 2$) Expand the LHS and arrange with x as the variable.	
<i>Value Substitution Method</i>	② Given $x^2 = a(x - 1)(x - 2) + b(x - 1) + c$, When $x = 1$, $1 = c \quad \dots \textcircled{1}$ When $x = 2$, $4 = b + c \quad \dots \textcircled{2}$ When $x = 3$, $9 = 2a + 2b + c \quad \dots \textcircled{3}$ From $\textcircled{1}$, $\textcircled{2}$, and $\textcircled{3}$, (Ans. $a = 1$, $b = 3$, $c = 1$) Substitute values of x to both sides of the equation.	J182b

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Topic	Formulas & Notes	Reference
	In order for $ax^2 + bx + c = 0$ to be an identity in x , the following must be true: $a = 0, b = 0, c = 0$.	J182b
	Given an equation, If we show that $\text{LHS} - \text{RHS} = 0$ Then we know that $\text{LHS} = \text{RHS}$	J188b
Proof of Equalities and Inequalities	Given an equation, If we show that $\text{LHS} - \text{RHS} > 0$ Then we know that $\text{LHS} > \text{RHS}$	J194a
	Given an equation, If we show that $\text{LHS} - \text{RHS} \geq 0$ Then we know that $\text{LHS} \geq \text{RHS}$	J194b
<i>Arithmetic Mean</i> <i>Geometric Mean</i>	Given $a > 0, b > 0$: $\frac{a + b}{2} \geq \sqrt{ab}$ [i.e., (arithmetic mean) $\geq$ (geometric mean)] The LHS equals the RHS when $a = b$.	J197b

[NOTES]

LEVEL K

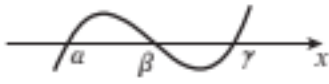
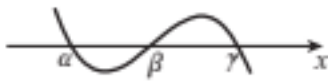
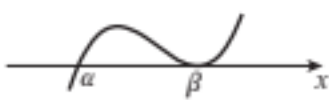
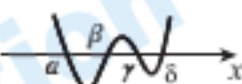
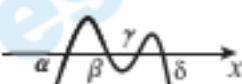
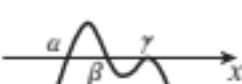
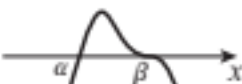
Topic	Formulas & Notes	Reference
Review of Linear Functions <i>Equation of a Linear Function</i>	$y = mx + b$ where m is the <i>slope</i> and b is the <i>y-intercept</i> . When $m > 0$  When $m < 0$ 	K3
<i>Slope of a Line</i>	Given two points with coordinates (x_1, y_1) and (x_2, y_2) , $m = \frac{y_2 - y_1}{x_2 - x_1} \quad \left(= \frac{\text{the change in } y}{\text{the change in } x} \right)$	K5a
Review of Quadratic Functions	The graph of a quadratic function is called a <i>parabola</i> . The parabola has a symmetrical axis called the <i>axis of the parabola</i> , or the <i>axis of symmetry</i> . The point of intersection of the axis of symmetry and the parabola is called the <i>vertex</i> of the parabola. 	K11b
<i>Equation of a Quadratic Function</i> <i>(a, b, c Format)</i> & <i>(p, q Format)</i>	- A function that can be written in the form: $y = ax^2 + bx + c$ (where $a \neq 0$) is called a <i>quadratic function</i>. - If a, p, and q are constants and $a \neq 0$, given a function of the form $y = a(x - p)^2 + q$ the <i>axis of symmetry</i> is $x = p$, and the coordinates of the <i>vertex</i> are (p, q).	K11a K13b
<i>Changing Format</i> <i>a, b, c to p, q</i>	To change the format of a quadratic equation from a, b, c to p, q , apply the method of <i>completing the square</i> . Ex. $y = x^2 - 2x + 3$ [Sol] $= (x^2 - 2x) + 3$ $= (x^2 - 2x + 1) - 1 + 3$ $= (x - 1)^2 + 2$	K14a
Quadratic Functions and Graphs <i>Translations</i>	The graph of $y = a(x - p)^2 + q$ is a <i>translation</i> of the graph of $y = ax^2$, p units along the x -axis, and q units along the y -axis.	K27b
Determining Equations of Quadratic Functions	When the graph of a parabola crosses the x -axis at points $(\alpha, 0)$ and $(\beta, 0)$ the equation of the function can be written as: $y = a(x - \alpha)(x - \beta)$	K35b

[NOTES]

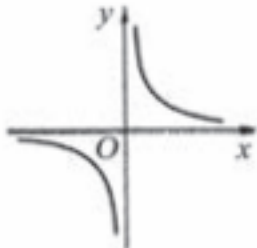
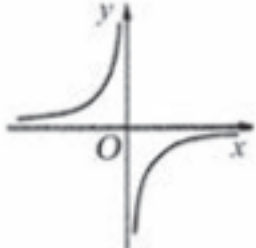
LEVEL K

Topic	Formulas & Notes	Reference																								
Maxima and Minima of Quadratic Functions 1	Given the quadratic function $y = ax^2 + bx + c$: - When $a > 0$, the parabola opens upward, and it has a minimum value at the vertex. (It does not have a maximum value.) The minimum value is $-\frac{b^2 - 4ac}{4a}$, at $x = -\frac{b}{2a}$. - When $a < 0$, the parabola opens downward, and it has a maximum value at the vertex. (It does not have a minimum value.) The maximum value is $-\frac{b^2 - 4ac}{4a}$, at $x = -\frac{b}{2a}$.	K41b																								
Maxima and Minima of Quadratic Functions 2	Given a quadratic function $y = ax^2 + bx + c$ with domain $\alpha \leq x \leq \beta$, (comparing the location of the specific domain to the location of the axis of symmetry, $x = -\frac{b}{2a}$), the maximum and minimum values of the graph $y = f(x)$ are defined at the left end of the domain, at the vertex, or at the right end of the domain.	K52 K55																								
Quadratic Functions and Equations	Given a quadratic function of the form $y = ax^2 + bx + c$, when the graph has <i>common points</i> with the x-axis at: 2 points, $D = b^2 - 4ac > 0$ 1 point, $D = b^2 - 4ac = 0$ 0 points, $D = b^2 - 4ac < 0$	K72b																								
Quadratic Functions and Inequalities	- The part of the graph of $y = ax^2 + bx + c$ at which $ax^2 + bx + c > 0$, consists of all the values of x at which the graph is above the x-axis. - The part of the graph of $y = ax^2 + bx + c$ at which $ax^2 + bx + c < 0$, consists of all the values of x at which the graph is below the x-axis.	K81a																								
	The relationship between the graph of the quadratic function $y = ax^2 + bx + c$, and the quadratic inequalities: $ax^2 + bx + c > 0, \quad ax^2 + bx + c \geq 0$ $ax^2 + bx + c < 0, \quad ax^2 + bx + c \leq 0$ can be summarized as follows. (Assume $a > 0$.) <table><tr><th>The sign of D</th><th>$D > 0$</th><th>$D = 0$</th><th>$D < 0$</th></tr><tr><td>The graph of $y = ax^2 + bx + c$</td><td></td><td></td><td></td></tr><tr><td>solving $ax^2 + bx + c > 0$</td><td>$x < \alpha, \beta < x$</td><td>$x < \alpha, \alpha < x$</td><td>All real numbers</td></tr><tr><td>solving $ax^2 + bx + c \geq 0$</td><td>$x \leq \alpha, \beta \leq x$</td><td>All real numbers</td><td>All real numbers</td></tr><tr><td>solving $ax^2 + bx + c < 0$</td><td>$\alpha < x < \beta$</td><td>No solution</td><td>No solution</td></tr><tr><td>solving $ax^2 + bx + c \leq 0$</td><td>$\alpha \leq x \leq \beta$</td><td>$x = \alpha$</td><td>No solution</td></tr></table>	The sign of D	$D > 0$	$D = 0$	$D < 0$	The graph of $y = ax^2 + bx + c$				solving $ax^2 + bx + c > 0$	$x < \alpha, \beta < x$	$x < \alpha, \alpha < x$	All real numbers	solving $ax^2 + bx + c \geq 0$	$x \leq \alpha, \beta \leq x$	All real numbers	All real numbers	solving $ax^2 + bx + c < 0$	$\alpha < x < \beta$	No solution	No solution	solving $ax^2 + bx + c \leq 0$	$\alpha \leq x \leq \beta$	$x = \alpha$	No solution	K86b
The sign of D	$D > 0$	$D = 0$	$D < 0$																							
The graph of $y = ax^2 + bx + c$																										
solving $ax^2 + bx + c > 0$	$x < \alpha, \beta < x$	$x < \alpha, \alpha < x$	All real numbers																							
solving $ax^2 + bx + c \geq 0$	$x \leq \alpha, \beta \leq x$	All real numbers	All real numbers																							
solving $ax^2 + bx + c < 0$	$\alpha < x < \beta$	No solution	No solution																							
solving $ax^2 + bx + c \leq 0$	$\alpha \leq x \leq \beta$	$x = \alpha$	No solution																							

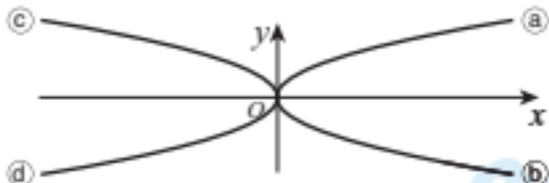
LEVEL K

Topic	Formulas & Notes	Reference
Quadratic Functions and Solutions of Quadratic Equations	Given a quadratic equation $ax^2 + bx + c = 0$ ($a > 0$) that has 2 solutions α, β ($\alpha < \beta$), you can find the signs of α, β by determining the signs of D, $-\frac{b}{2a}$, and $f(0)$. - When $D > 0$, $-\frac{b}{2a} > 0$, $f(0) > 0$, then $\alpha > 0, \beta > 0$. - When $D > 0$, $-\frac{b}{2a} < 0$, $f(0) > 0$, then $\alpha < 0, \beta < 0$. - When $D > 0$, $f(0) < 0$, then $\alpha < 0, \beta > 0$.	K91b
Higher Degree Functions <i>Rough Graphs of Cubic Functions</i>	$y = a(x - \alpha)(x - \beta)(x - \gamma)$, (where $\alpha < \beta < \gamma$): - When $a > 0$  - When $a < 0$  $y = a(x - \alpha)(x - \beta)^2$, (where $\alpha < \beta$): - When $a > 0$  - When $a < 0$  $y = a(x - \alpha)^3$: - When $a > 0$  - When $a < 0$ 	K104b
<i>Rough Graphs of Quartic Functions</i>	$y = a(x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$ ($\alpha < \beta < \gamma < \delta$): - When $a > 0$  - When $a < 0$  $y = a(x - \alpha)(x - \beta)(x - \gamma)^2$ ($\alpha < \beta < \gamma$): - When $a > 0$  - When $a < 0$  $y = a(x - \alpha)^2(x - \beta)^2$ ($\alpha < \beta$): - When $a > 0$  - When $a < 0$  $y = a(x - \alpha)(x - \beta)^3$ ($\alpha < \beta$): - When $a > 0$  - When $a < 0$  $y = a(x - \alpha)^4$: - When $a > 0$  - When $a < 0$ 	K107b

LEVEL K

Topic	Formulas & Notes	Reference
Graphs of Fractional Functions 1	On the graph of $y = \frac{a}{x}$ (where $a \neq 0$) the <i>asymptotes</i> are: the y-axis and the x-axis When $a > 0$  When $a < 0$ 	K122
<i>Asymptotes and Translations</i>	Given a fractional function of the general form $y = \frac{k}{x - p} + q$ ($k \neq 0$), the equations of the graph's asymptotes are $x = p, y = q$. This graph has been translated from the graph of $y = \frac{k}{x}$, p units along the x-axis, and q units along the y-axis.	K124a K127b
<i>Identifying Asymptotes</i>	In order to identify the asymptotes easily, you can transform an equation as follows: $y = \frac{x + 1}{x - 2} = \frac{(x - 2) + 3}{x - 2} = 1 + \frac{3}{x - 2}$	K125a
<i>x-intercept</i>	To find the x-coordinate of the point of intersection of the graph $y = \frac{ax + b}{cx + d}$ and the x-axis, you can solve the equation $ax + b = 0$ (which is the numerator = 0). (For example, given $y = \frac{x + 1}{x - 2}$, solving the equation $x + 1 = 0$ we get $x = -1$, which is the <i>x-intercept</i>, i.e. the x-coordinate of the x-axis point of intersection.)	K125b
Graphs of Fractional Functions 2 <i>Asymptotes</i>	The equations of the asymptotes of the graph $y = ax + \frac{b}{x}$ are: $x = 0, y = ax$	K137a
Fractional Equations and Inequalities	When solving fractional equations, we must always check each solution and exclude any one that makes the denominator of the original equation 0. $A = B \Leftrightarrow AC = BC$ ($AC = BC \Leftrightarrow A = B$ or $C = 0$)	K144
Graphs of Irrational Functions	- An irrational function of the form $y = \sqrt{k(x - p)} + q$ is a translation of $y = \sqrt{kx}$, p units along the x-axis, and q units along the y-axis. - An irrational function of the form $y = -\sqrt{k(x - p)} + q$ is a translation of $y = -\sqrt{kx}$, p units along the x-axis, and q units along the y-axis.	K157b

LEVEL K

Topic	Formulas & Notes	Reference
<i>Domain & Range</i> <i>Translations</i> <i>Rough Graphs</i>	- Given an irrational function of the general form $y = \sqrt{k(x - p)} + q$, the domain and range of the function are as follows: When $k > 0$, Domain: $x \geq p$, Range: $y \geq q$ When $k < 0$, Domain: $x \leq p$, Range: $y \geq q$ - An irrational function of the general form $y = \sqrt{k(x - p)} + q$ is a translation of the graph of $y = \sqrt{kx}$, p units along the x-axis, and q units along the y-axis. - The rough graphs of some common irrational functions are shown below. a $y = \sqrt{x}$, b $y = -\sqrt{x}$, c $y = \sqrt{-x}$, d $y = -\sqrt{-x}$ 	K160b
Irrational Equations and Inequalities	When solving irrational equations we must always check the solutions by using either of the following two methods: - The solution is the x-coordinate of the common point on the graph. OR, - When substituting the solution into the original equation, LHS = RHS. $A = B \Leftrightarrow A^2 = B^2 \quad (A^2 = B^2 \Leftrightarrow A = B \text{ or } A = -B)$	K164b
Exponential Functions <i>Definition of a^0 and a^{-n}</i>	- If $a^m \times a^n = a^{m+n}$ is true even when $m = 0$, then, $a^0 \times a^n = a^n$ so, $a^0 = 1$. - If $a^m \div a^n = a^{m-n}$ is true even when $m = 0$, then, $1 \div a^n = a^{-n}$ so, $a^{-n} = \frac{1}{a^n}$.	K171a
<i>Laws of Exponents</i>	When $a \neq 0$, $b \neq 0$, $a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$ $a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$	K171b
<i>The n^{th} Root of a</i>	The number which when raised to the power of n is equal to a is called the n^{th} root of a. ($\sqrt[3]{8}$ is called the 3rd root of 8, $\sqrt[3]{-8}$ is the 3rd root of -8, $\sqrt[n]{a}$ is the n^{th} root of a.)	K173b

LEVEL K

Topic	Formulas & Notes	Reference
<i>The Number of n^{th} Roots</i>	- When n is an even number (square roots, 4th roots . . .), When $a > 0$, there are 2 n^{th} roots (ex. $\sqrt[n]{a}$, $-\sqrt[n]{a}$). When $a < 0$, there are no real number n^{th} roots. - When n is an odd number (cube roots, 5th roots . . .), There is only 1 real number n^{th} root. (However, if we include imaginary numbers, there are 3 cube roots, 4 fourth roots, etc.)	K174a
<i>Laws of Exponents</i>	When $a > 0$, $b > 0$ and m, n, and p are positive integers: $\sqrt[n]{a^m} = (\sqrt[n]{a})^m \quad \sqrt[m]{a^{np}} = \sqrt[m]{a^n} \quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$	K174b
	$\sqrt[3]{-16} = -\sqrt[3]{16}, \quad \sqrt[5]{-32} = -\sqrt[5]{32} = -2$ When n is an odd number, $\sqrt[n]{-a} = -\sqrt[n]{a}$ (where $a > 0$).	K176a
<i>Laws of Exponents</i>	When $a > 0$, m is an integer, and n is a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \left(a^{\frac{1}{n}} = \sqrt[n]{a} \right)$	K177a
Graphs of Exponential Functions	When $a > 0$ and $a \neq 1$, a function of the form $y = a^x$ is called an <i>exponential function</i> of x, where a is the <i>base</i>. The graph of $y = a^x$ passes through point $(0, 1)$, and the x-axis is the asymptote. When $0 < a < 1$  When $1 < a$ 	K182b
<i>Translations</i>	The graph of $y = a^{x-p} + q$ (where $a > 0$ and $a \neq 1$) is a translation of the graph of $y = a^x$, p units along the x-axis, and q units along the y-axis.	K185b
<i>Comparing Numbers</i>	- Given the exponential function $y = a^x$, when $a > 1$, as x increases, y increases. Therefore, $p < q \Leftrightarrow a^p < a^q \quad (p < q \text{ if and only if } a^p < a^q)$ - Given the exponential function $y = a^x$, when $0 < a < 1$, as x increases, y decreases. Therefore, $p < q \Leftrightarrow a^p > a^q \quad (p < q \text{ if and only if } a^p > a^q)$ - Given the exponential functions $y = a^x$ and $y = b^x$, when $a > 0$, $b > 0$, and $p > 0$, $a < b \Leftrightarrow a^p < b^p \quad (a < b \text{ if and only if } a^p < b^p)$	K187a K187b K188a

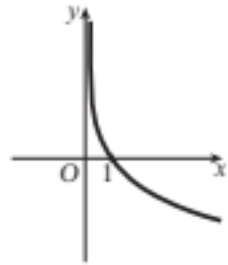
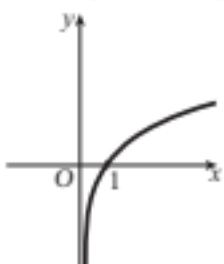
LEVEL K

Topic	Formulas & Notes	Reference
Exponential Equations and Inequalities <i>Solving Methods</i>	• If we let $a^x = X$ (where $X > 0$) and can rewrite the equation in terms of X, then we solve for X. • When $a^x = a^m$, then $x = m$	K194a
	• When $a > 1$, then $a^x > a^m \Leftrightarrow x > m$ • When $0 < a < 1$, then $a^x > a^m \Leftrightarrow x < m$	K196

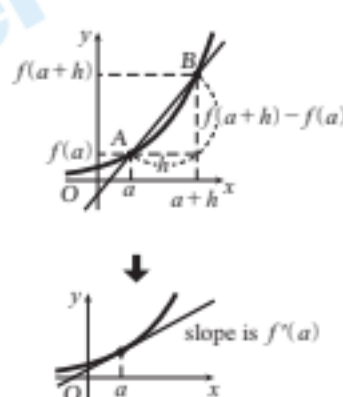
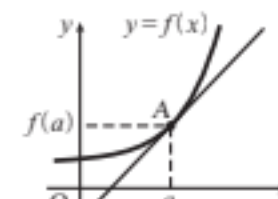
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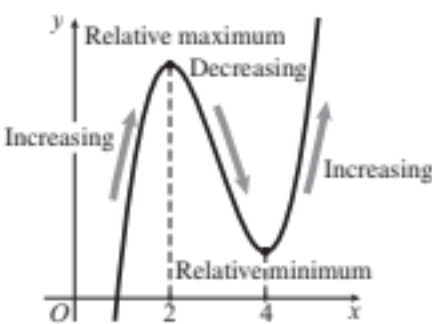
LEVEL L

Topic	Formulas & Notes	Reference
Logarithmic Functions	$a^n = N$ can be written as $\log_a N = n$	L1a
	Given $\log_a N$ (where $a > 0, a \neq 1, N > 0$): a is called the <i>base</i> , and N is called the <i>antilogarithm</i> . $\log_a N$ is called the <i>logarithm of N, to the base a</i> .	L1b
<i>Properties of Logarithms</i>	When $a > 0, a \neq 1, M > 0, N > 0$, 1 $\log_a 1 = 0, \log_a a = 1, \log_a a^m = m$ 2 $\log_a MN = \log_a M + \log_a N$ 3 $\log_a \frac{M}{N} = \log_a M - \log_a N, \left[\log_a \frac{1}{N} = -\log_a N \right]$ 4 $\log_a M^n = n \log_a M \quad \left[\log_a \sqrt[n]{M} = \frac{1}{n} \log_a M \right]$	L5a
Graphs of Logarithmic Functions <i>Logarithmic Base Conversion Formula</i>	When $\log_a M = x, a^x = M$ Taking the logarithm, to the base b , of both sides, $\log_b a^x = \log_b M$ $x \log_b a = \log_b M$ $x = \frac{\log_b M}{\log_b a}$ Therefore, $\log_a M = \frac{\log_b M}{\log_b a}$ (where $a > 0, b > 0, M > 0$ and $a \neq 1, b \neq 1$)	L11a
	$\log_a b = \frac{1}{\log_b a} \qquad \log_a b \cdot \log_b a = 1$	L11b L13a
	$a^{\log_a M} = M$	L14b
	When $a > 0$ and $a \neq 1$, a function of the form $y = \log_a x$ is called the <i>logarithmic function of x</i>, where a is the base. The graph of the logarithmic function $y = \log_a x$ passes through point $(1, 0)$, and its asymptote is the y-axis. When $0 < a < 1$  When $1 < a$ 	L16b
<i>Translations</i>	The graph of the logarithmic function $y = \log_a(x - p) + q$ (where $a > 0$ and $a \neq 1$) is a translation of the graph of $y = \log_a x$, p units along the x -axis, and q units along the y -axis.	L17b

LEVEL L

Topic	Formulas & Notes	Reference
	Given the logarithmic functions $y = \log_a p$, $y = \log_a q$, When $a > 1$, then $p < q \Leftrightarrow \log_a p < \log_a q$. When $0 < a < 1$, then $p < q \Leftrightarrow \log_a p > \log_a q$.	L19b
Logarithmic Equations and Inequalities <i>Solving Methods</i>	- If the initial equation can be written in the form $\log_a A = \log_a B$, we can equate the antilogarithms, $A = B$. - If we let $\log_a x = X$ and can rewrite the equation in terms of X, then we solve for X. When $\log_a x = b$, then $x = a^b$. - We check all solutions by considering that: antilogarithms are > 0, the base is > 0, and the base $\neq 1$	L22 L23
	When $a > 1$, $\log_a x > b \Leftrightarrow x > a^b$ When $0 < a < 1$, $\log_a x > b \Leftrightarrow 0 < x < a^b$	L24
Modulus Functions	a is called the <i>absolute value</i> of a. When a is positive or zero, $a = a$. When a is negative, $a = -a$.	L31a
Limits and Derivatives	The <i>average rate of change</i> of $y = f(x)$ when x varies from $x = a$ to $x = b$ is expressed as: $\frac{f(b) - f(a)}{b - a}$	L43a
<i>Differential Coefficient</i>	Given $y = f(x)$, the average rate of change, when x varies from a to $a + h$, can be expressed as $\frac{f(a + h) - f(a)}{h}$. In this case, $f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$ $f'(a)$ is the <i>differential coefficient</i> for $x = a$. ($f'(a)$ indicates the slope of the line that is tangent to the graph of $y = f(x)$ at $x = a$.) 	L44a
<i>Derivative of f(x)</i>	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$	L46a
	When n is a natural number, $(x^n)' = nx^{n-1}$.	L47b
<i>Rules of Differentiation</i>	If $y = k$, then $y' = 0$ (where k is a constant) If $y = kf(x)$, then $y' = kf'(x)$ (where k is a constant) If $y = f(x) + g(x)$, then $y' = f'(x) + g'(x)$ If $y = f(x) - g(x)$, then $y' = f'(x) - g'(x)$ If $y = f(x)g(x)$, then $y' = f'(x)g(x) + f(x)g'(x)$	L48a
		L49b
Tangent Lines	The slope of the tangent line at point $(a, f(a))$, on the curve $y = f(x)$, is $f'(a)$. The equation of the tangent line at point $(a, f(a))$ is as follows: $y - f(a) = f'(a)(x - a)$ 	L51a

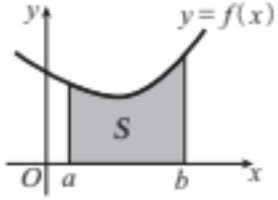
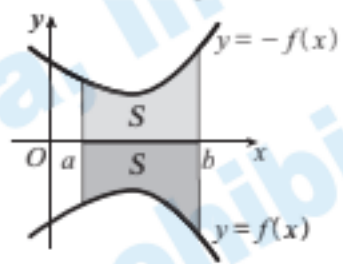
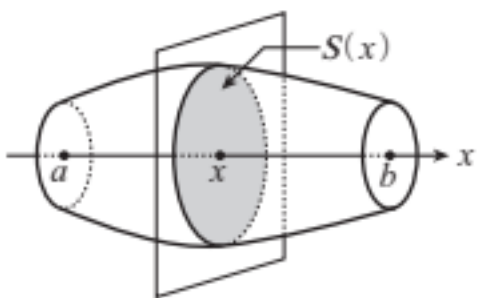
LEVEL L

Topic	Formulas & Notes	Reference
Relative Maxima and Minima 1	The cubic function $f(x) = x^3 - 9x^2 + 24x - 15$ is: • increasing when $x < 2$ • decreasing when $2 < x < 4$ • increasing when $4 < x$ And, • At $x = 2$ the function has a relative maximum. • At $x = 4$ the function has a relative minimum. 	L61a
	We do not identify that a function has a relative maximum or a relative minimum until we find out the variation around the point where $y' = 0$ (or $f'(x) = 0$). 	L62b
Variation Table	• When $y' > 0$ we use +, when $y' < 0$ we use - • ↗ means increasing, ↘ means decreasing • There is a relative maximum value when the arrow changes from ↗ to ↘, and there is a relative minimum value when the arrow changes from ↘ to ↗.	L62b
	For all values of x, When $y' > 0$, y is always increasing, and thus there is no relative extreme value. When $y' < 0$, y is always decreasing, and thus there is no relative extreme value.	L68a
Relative Maxima and Minima 2	When the coefficient of x^3 is a letter, as in $ax^3 + 2ax^2 + ax + 1$, we find the relative extreme values by analyzing 2 cases (a) when $a > 0$ and (b) when $a < 0$.	L73b
Maxima and Minima 1	To find a maximum and a minimum value of a function $y = f(x)$ on an interval $a \leq x \leq b$, (a) We create a variation table on the interval $a \leq x \leq b$. (b) We determine and then compare the relative extreme values and the values of $f(a)$ and $f(b)$ on both ends of the interval.	L82b
	In exercises where we have to find the minimum value, when there is no relative minimum value in the interval, we obtain the minimum value either at the right end or the left end of the domain. In the exercises where we have to find the maximum value, when there is no relative maximum value in the interval, we obtain the maximum value either at the right end or the left end of the domain.	L83b L84b

LEVEL L

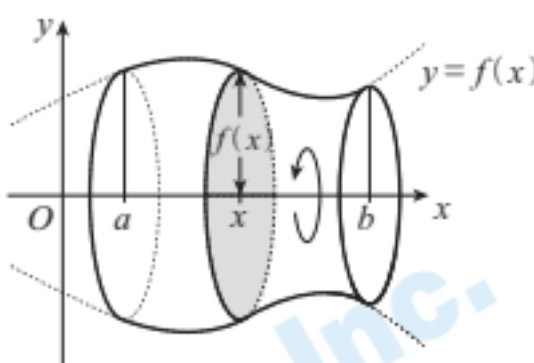
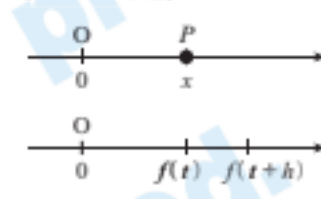
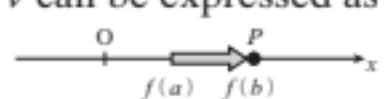
Topic	Formulas & Notes	Reference
Applications to Equations and Inequalities	Real roots of an equation $f(x) = 0$ will be the x -coordinates of common points where the graph of the function $y = f(x)$ and the x -axis cross.	L101a
Indefinite and Definite Integrals	When one of the indefinite integrals of $f(x)$ is $F(x)$, $\int f(x) dx = F(x) + C \quad (C \text{ is the constant of integration.})$	L111b
<i>Formula for Indefinite Integrals</i>	When n is a positive integer, or zero, $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad (C \text{ is the constant of integration.})$	L111b
<i>Properties of Indefinite Integrals</i>	$\int kf(x) dx = k \int f(x) dx \quad (k \text{ is a constant.})$ $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$ $\int [f(x) - g(x)] dx = \int f(x) dx - \int g(x) dx$	L112a
<i>Definition of Definite Integral</i>	When one of the indefinite integrals of $f(x)$ is $F(x)$, $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$	L117b
Definite Integrals 1 <i>Properties of Definite Integrals</i>	$\int_a^b kf(x) dx = k \int_a^b f(x) dx \quad (k \text{ is a constant.})$ $\int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx$ $\int_a^b [f(x) - g(x)] dx = \int_a^b f(x) dx - \int_a^b g(x) dx$	L121a
<i>Property of Definite Integrals</i>	$\int_{-a}^a x^n dx = \begin{cases} 0 & (\text{when } n \text{ is } 1, 3, 5, \dots) \\ 2 \int_0^a x^n dx & (\text{when } n \text{ is } 0, 2, 4, \dots) \end{cases}$	L123b
<i>Property of Definite Integrals</i>	When $\alpha < \beta$, $\int_{\alpha}^{\beta} (x - \alpha)(x - \beta) dx = -\frac{1}{6}(\beta - \alpha)^3$	L124b
<i>Properties of Definite Integrals</i>	$\int_a^b f(x) dx = - \int_b^a f(x) dx$ $\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$	L126b

LEVEL L

Topic	Formulas & Notes	Reference
Definite Integrals 2	$\frac{d}{dx} \int_a^x f(t) dt = f(x)$ (where a is a constant)	L131b
Areas 1	Generally when $f(x) \geq 0$ we can find the area, S, of the region enclosed by the curve $y = f(x)$ and the x-axis in the interval of $[a, b]$ by using the following formula: $S = \int_a^b f(x) dx$ 	L141b
	In the interval $[a, b]$ there are two regions, one enclosed by $y = f(x)$ and the x-axis and one enclosed by $y = -f(x)$ and the x-axis, that are symmetric with respect to the x-axis and equal in area. Therefore, when $f(x) \leq 0$ we can find the area by using the following formula: $S = - \int_a^b f(x) dx$ 	L142a
	The area, S, of the region enclosed by two curves, $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$ in the interval $[a, b]$ is: $S = \int_a^b [f(x) - g(x)] dx \quad (\text{where } f(x) \geq g(x))$	L146a
<i>Area of the Region Involving a Parabola</i>	$\int_{\alpha}^{\beta} (x - \alpha)(x - \beta) dx = -\frac{1}{6}(\beta - \alpha)^3$	L148a
Volumes	If $S(x)$ is a function of x representing the cross-sectional area of the region on a given solid cut by a plane which is perpendicular to the x-axis, then the volume for values where $a \leq x \leq b$ is given by the following formula: $V = \int_a^b S(x) dx$ 	L161a

[NOTES]

LEVEL L

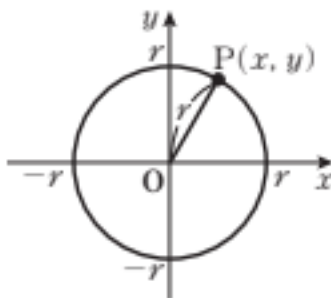
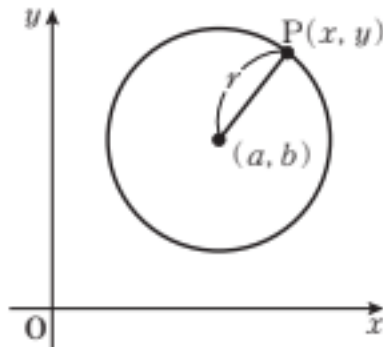
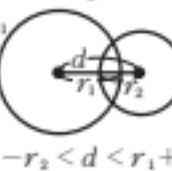
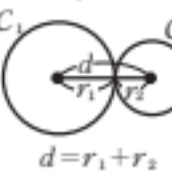
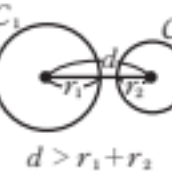
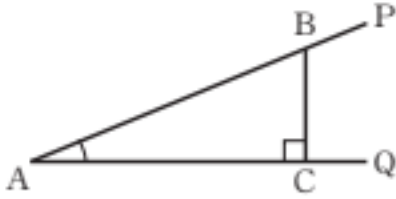
Topic	Formulas & Notes	Reference
Volume of a Solid of Revolution	The solid that forms by rotating a line or curve around an axis is called a <i>solid of revolution</i>. We let V be the volume of the solid that forms by rotating the region enclosed by the function $y = f(x)$, the x-axis, line $x = a$, and line $x = b$. Since the surface, which is cut by a plane perpendicular to the x-axis, would be a circle, its area is $S(x) = \pi y^2 = \pi[f(x)]^2$ Therefore, the volume, V, of the solid of revolution for values where $a \leq x \leq b$ is: $V = \int_a^b \pi y^2 dx = \pi \int_a^b [f(x)]^2 dx$ 	L165a
Velocity and Distance	When a point P moves on a number line, if the position of the point P at time t is expressed as the x-coordinate, then x is a function of t. Letting this function be $x = f(t)$, the <i>average velocity</i> of point P from time t to time $t + h$ is expressed as: $\frac{f(t + h) - f(t)}{h}$ 	L171a
Velocity	When the position at time t, of point P moving on a number line, is expressed as $x = f(t)$, we can express the <i>velocity</i> of point P as: $v = \frac{dx}{dt} = \lim_{h \rightarrow 0} \frac{f(t + h) - f(t)}{h} = f'(t)$	L171b
Displacement	Given a point P moving on a number line, if we let the position at time t be $x = f(t)$, the velocity v can be expressed as $v = \frac{dx}{dt} = f'(t).$  The <i>displacement</i> (the change in position) of point P from $t = a$ to $t = b$ is expressed as follows: $f(b) - f(a) = \int_a^b f'(t) dt = \int_a^b v dt$	L175a
Distance	Generally, given a point P moving at time t with velocity v, the distance, s, traveled from time $t = a$ to $t = b$ is expressed by the following formula: $s = \int_a^b v dt$	L176b

LEVEL M

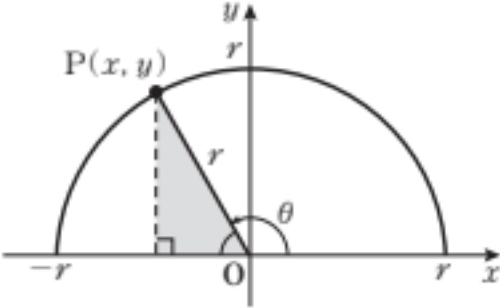
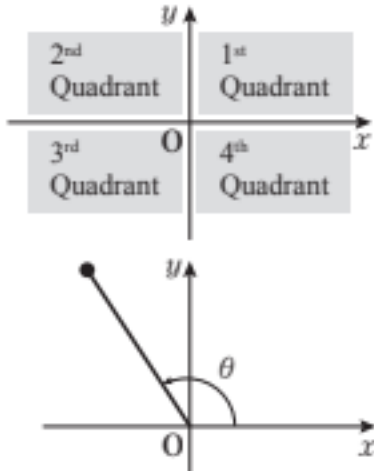
Topic	Formulas & Notes	Reference
Points and Lines 1 <i>Distance Formula</i>	The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Also, the distance between origin O and point $A(x_1, y_1)$ is $OA = \sqrt{x_1^2 + y_1^2}$	M1b
<i>Coordinates of Internal/External Dividing Points</i>	Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the points that divide line segment AB in the ratio $m : n$ are as follows: Internally, $\left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n} \right)$ Externally, $\left(\frac{-nx_1 + mx_2}{m - n}, \frac{-ny_1 + my_2}{m - n} \right)$	M6b
<i>Midpoint</i>	The coordinates of the midpoint M of line segment AB , where $A(x_1, y_1)$ and $B(x_2, y_2)$ are given by $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	M7a
<i>Center of Gravity of Triangles</i>	Given $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, the coordinates of the center of gravity G are given by $G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$	M8b
Points and Lines 2 <i>Equation of a Line I</i>	The equation of a line passing through point and (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$	M11a
<i>Parallel Condition</i>	For two lines $y = m_1x + n_1$ and $y = m_2x + n_2$, the parallel condition is $m_1 = m_2$.	M14a
<i>Perpendicular Condition</i>	For two lines $y = m_1x + n_1$ and $y = m_2x + n_2$, the perpendicular condition is $m_1m_2 = -1$.	M15b
Points and Lines 3 <i>Distance from a Point to a Line</i>	The distance d from point (x_1, y_1) to line $ax + by + c = 0$ is $d = \frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$	M25b

[NOTES]

LEVEL M

Topic	Formulas & Notes	Reference
Circles 1 <i>Equation of a Circle I</i>	The equation of a circle with center at origin O and radius r is $x^2 + y^2 = r^2$ 	M31a
<i>Equation of a Circle II</i>	The equation of a circle with center at point (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$ 	M31b
<i>Positional Relationship between a Line and a Circle</i>	When the quadratic equation $ax^2 + bx + c = 0$ is obtained after y is eliminated from each equation of a line and a circle, let the discriminant be $D (= b^2 - 4ac)$. $D > 0 \Leftrightarrow$ They intersect at two different points. $D = 0 \Leftrightarrow$ They are tangent and intersect at only one point. $D < 0 \Leftrightarrow$ They do not intersect.	M37a
Circles 2 <i>Tangent to a Circle</i>	The equation of the tangent to circle $x^2 + y^2 = r^2$ at point $P(x_1, y_1)$ on the circle is $x_1x + y_1y = r^2$	M41b
<i>Positional Relationship between Two Circles</i>	Let the radii of two circles C_1 and C_2 be r_1 and r_2 ($r_1 > r_2$) respectively and the distance between the centers of the circles be d. [1] Completely inside  $d < r_1 - r_2$ [2] Internally tangent  $d = r_1 - r_2$ [3] Intersect at two points  $r_1 - r_2 < d < r_1 + r_2$ [4] Externally tangent  $d = r_1 + r_2$ [5] Completely outside  $d > r_1 + r_2$ ※ [3]~[5] are also true when $r_1 = r_2$. If it is not possible to define which is greater between the values of r_1 and r_2, use $r_1 - r_2$ instead of $r_1 - r_2$ in [1]~[3].	M49a
Trigonometric Ratios 1	Given $\angle PAQ$, drop a perpendicular BC from point B on AP to AQ and form a right-angled triangle ABC. When A represents the size of $\angle A$,  $\frac{BC}{AB}$ is called the sine of A and expressed as sin A. $\frac{AC}{AB}$ is called the cosine of A and expressed as cos A. $\frac{BC}{AC}$ is called the tangent of A and expressed as tan A.	M81a

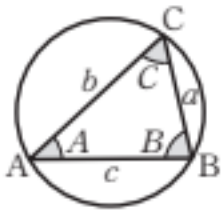
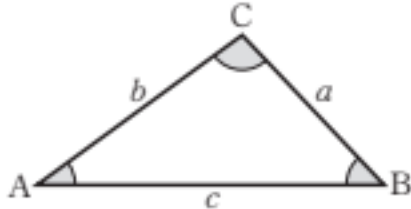
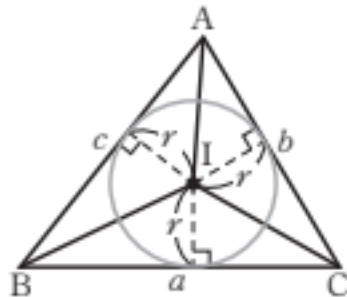
LEVEL M

Topic	Formulas & Notes	Reference
<i>Trigonometric Identities I</i>	$\tan A = \frac{\sin A}{\cos A}$ $\sin^2 A + \cos^2 A = 1$	M86a
<i>Trigonometric Identity II</i>	$1 + \tan^2 A = \frac{1}{\cos^2 A}$	M87b
Trigonometric Ratios 2 <i>Trigonometric Ratios of $0^\circ \leq \theta \leq 180^\circ$</i>	$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$ 	M91b
<i>Trigonometric Ratios of $90^\circ - \theta$</i>	$\sin(90^\circ - \theta) = \cos \theta$ $\cos(90^\circ - \theta) = \sin \theta$ $\tan(90^\circ - \theta) = \frac{1}{\tan \theta}$	M98a
<i>Trigonometric Ratios of $180^\circ - \theta$</i>	$\sin(180^\circ - \theta) = \sin \theta$ $\cos(180^\circ - \theta) = -\cos \theta$ $\tan(180^\circ - \theta) = -\tan \theta$	M98b
Properties of Trigonometric Functions 1 <i>Definitions of Trigonometric Functions</i>	$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \tan \theta = \frac{y}{x}$	M101a
<i>Trigonometric Functions of $\theta + 360^\circ \times n$</i>	$\sin(\theta + 360^\circ \times n) = \sin \theta$ $\cos(\theta + 360^\circ \times n) = \cos \theta$ $\tan(\theta + 360^\circ \times n) = \tan \theta$	M102b
<i>Trigonometric Functions of $-\theta$</i>	$\sin(-\theta) = -\sin \theta$ $\cos(-\theta) = \cos \theta$ $\tan(-\theta) = -\tan \theta$	M104b
<i>Trigonometric Functions of $\theta + 180^\circ$</i>	$\sin(\theta + 180^\circ) = -\sin \theta$ $\cos(\theta + 180^\circ) = -\cos \theta$ $\tan(\theta + 180^\circ) = \tan \theta$	M105b
<i>Trigonometric Functions of $\theta + 90^\circ$</i>	$\sin(\theta + 90^\circ) = \cos \theta$ $\cos(\theta + 90^\circ) = -\sin \theta$ $\tan(\theta + 90^\circ) = -\frac{1}{\tan \theta}$	M106b
	The four regions on a coordinate plane are called the 1st Quadrant, 2nd Quadrant, 3rd Quadrant and 4th Quadrant. When the terminal side of θ is in the 2nd Quadrant, θ is called an angle in the 2nd Quadrant. 	M107a

LEVEL M

Topic	Formulas & Notes	Reference
Properties of Trigonometric Functions 2 <i>Circular Measure</i>	Since $180^\circ = \pi$ radians, $1^\circ = \frac{\pi}{180} \text{ radians, } 1 \text{ radian} = \frac{180^\circ}{\pi}$	M111a
Graphs of Trigonometric Functions	Let the point of intersection of the terminal side of θ and the unit circle be P. The y-coordinate of P is $\sin\theta$. The x-coordinate of P is $\cos\theta$. Using these, we can draw the graphs of $y = \sin\theta$ and $y = \cos\theta$. Let the point of intersection of line OP and line $x = 1$ be T(1, m). Then $\tan\theta = m$ is true. Using this, we can draw the graph of $y = \tan\theta$.	M131a M132a M137a
Graph of $y = \sin\theta$		M131a
Graph of $y = \cos\theta$		M132a
Graph of $y = \tan\theta$		M137a
Addition Formulas 1 <i>Addition Formulas I</i>	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	M152a
<i>Addition Formulas II</i>	$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$	M155b

LEVEL M

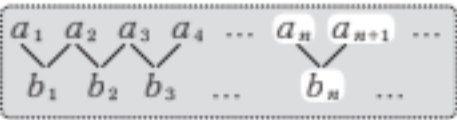
Topic	Formulas & Notes	Reference
Addition Formulas 2 <i>Double-Angle Formulas</i>	$\sin 2\alpha = 2 \sin \alpha \cos \alpha$ $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $\quad = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$ $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$	M161b
<i>Triple-Angle Formulas</i>	$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$ $\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$	M163b
<i>Half-Angle Formulas</i>	$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}, \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}, \tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$	M164b
Addition Formulas 3 <i>Conversion of $a \sin \theta + b \cos \theta$</i>	$a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2} \sin(\theta + \alpha)$ where $\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}, \sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$	M171b
<i>Product-to-Sum Formulas</i>	$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$ $\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$ $\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$ $\sin \alpha \sin \beta = -\frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)]$	M175b
<i>Sum-to-Product Formulas</i>	$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$ $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$ $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$	M177b
Laws of Sines and Cosines <i>Law of Sines</i>	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (R is the radius of the circumscribed circle of $\triangle ABC$) 	M182a
<i>Law of Cosines</i>	Given $\triangle ABC$, $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = c^2 + a^2 - 2ca \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$ 	M184b
Area of Triangles Formula	$S = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C$	M191a
<i>Area of a Triangle with an Inscribed Circle</i>	Let S be the area of $\triangle ABC$, I be the center of the inscribed circle, and r be the radius. $S = \triangle IBC + \triangle ICA + \triangle IAB$ $\quad = \frac{1}{2} ar + \frac{1}{2} br + \frac{1}{2} cr$ $\quad = \frac{1}{2} r(a + b + c)$ 	M196a

LEVEL M

[NOTES]

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LEVEL N

Topic	Formulas & Notes	Reference
Arithmetic Sequences <i>General Term of an Arithmetic Sequence</i>	A sequence whose terms are found by successively adding a fixed number d to the 1st term a is called an arithmetic sequence (or arithmetic progression). d is called the common difference of the arithmetic sequence. The general term of an arithmetic sequence $\{a_n\}$ with 1st term a and common difference d is $a_n = a + (n-1)d$	N2a N2b
<i>Sum of an Arithmetic Sequence</i>	Let S_n be the sum of an arithmetic sequence with 1st term a, common difference d, last term l and number of terms n. $S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$	N6b
Geometric Sequences <i>General Term of a Geometric Sequence</i>	A sequence whose terms are found by successively multiplying a fixed number r to the 1st term a is called a geometric sequence (or geometric progression). r is called the common ratio of the geometric sequence. The general term of a geometric sequence $\{a_n\}$ with 1st term a and common ratio r is $a_n = ar^{n-1}$	N11a N12a
<i>Sum of a Geometric Sequence</i>	Let S_n be the sum of a geometric sequence with 1st term a, common ratio r and number of terms n. When $r \neq 1$, $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$ When $r = 1$, $S_n = na$ When $r = 1$, S_n is the sum of n terms of a.	N16b
Various Sequences 1 <i>Summation Formulas I</i>	$\sum_{k=1}^n k = \frac{1}{2}n(n+1), \quad \sum_{k=1}^n c = nc \quad (c \text{ is a constant})$	N22a
<i>Summation Properties</i>	$\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k, \quad \sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k \quad (c \text{ is a constant})$	N22b
<i>Summation Formula II</i>	$\sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$	N23b
<i>Summation Formula III</i>	$\sum_{k=1}^n k^3 = \left[\frac{1}{2}n(n+1) \right]^2$	N24b
<i>Summation Formula IV</i>	$\sum_{k=1}^n ar^{k-1} = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1} \quad (r \neq 1)$	N25a
Various Sequences 2	The term which is derived by taking the difference between two consecutive terms in the sequence $\{a_n\}$ is $b_n = a_{n+1} - a_n \quad (n=1, 2, 3, \dots)$  Generally, the sequence $\{b_n\}$ is called the sequence of differences of $\{a_n\}$.	N31b

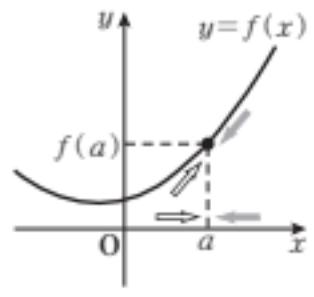
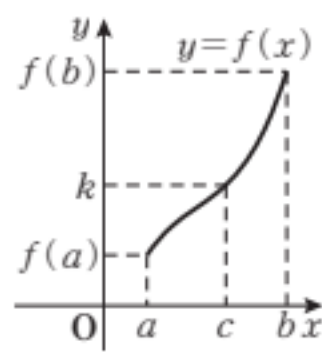
LEVEL N

Topic	Formulas & Notes	Reference
<i>Sequence of Differences and General Term</i>	Let $\{b_n\}$ be the sequence of differences of the sequence $\{a_n\}$. When $n \geq 2$, $a_n = a_1 + \sum_{k=1}^{n-1} b_k$	N32a
<i>Sum of a Sequence and General Term</i>	Let S_n be the sum of the first n terms of the sequence $\{a_n\}$. The 1 st term a_1 is $a_1 = S_1$. When $n \geq 2$, $a_n = S_n - S_{n-1}$	N34a
Recurrence Relations	Given that the sequence $\{a_n\}$ satisfies the following two conditions: (i) $a_1 = 2$ (ii) $a_{n+1} = 3a_n - 1$ ($n = 1, 2, 3, \dots$) When each term is successively determined from $a_2, a_3, a_4, \dots$ one after another, then one sequence $\{a_n\}$ will be defined. A condition such as (ii), where the expression defines the relationship between two or more consecutive terms in a sequence, is called a recurrence relation .	N41a
Mathematical Induction	To prove that a proposition P is true for all natural numbers n by mathematical induction, the following two statements have to be proved. (i) P is true when $n = 1$. (ii) If P is true when $n = k$, then P is also true when $n = k + 1$.	N51a
Infinite Sequences	A sequence with infinite terms $a_1, a_2, a_3, \dots, a_n, \dots$ is called an infinite sequence , and it is expressed as $\{a_n\}$.	N61a
	When $\{a_n\}$ diverges to positive (or negative) infinity , the limit is positive (or negative) infinity, and is expressed as $\lim_{n \rightarrow \infty} a_n = +\infty \quad \lim_{n \rightarrow \infty} a_n = -\infty$ Divergent sequences which do not diverge to positive or negative infinity are said to oscillate .	N62a
<i>Limit of Sequence</i>	$\left\{ \begin{array}{l} \text{Converges} \quad \lim_{n \rightarrow \infty} a_n = \alpha \quad (\text{converges to a constant value } \alpha) \quad \dots \textcircled{1} \\ \text{Diverges} \quad \left\{ \begin{array}{l} \lim_{n \rightarrow \infty} a_n = \infty \quad (\text{diverges to positive infinity}) \quad \dots \textcircled{2} \\ \lim_{n \rightarrow \infty} a_n = -\infty \quad (\text{diverges to negative infinity}) \quad \dots \textcircled{3} \\ \text{Oscillates} \quad (\text{no limit}) \quad \dots \textcircled{4} \end{array} \right. \end{array} \right.$	N62b
<i>Properties of Limits of Sequences</i>	When the sequences $\{a_n\}$ and $\{b_n\}$ converge, where $\lim_{n \rightarrow \infty} a_n = \alpha$ and $\lim_{n \rightarrow \infty} b_n = \beta$, $\lim_{n \rightarrow \infty} k a_n = k \alpha \quad (k \text{ is a constant})$ $\lim_{n \rightarrow \infty} (a_n + b_n) = \alpha + \beta, \quad \lim_{n \rightarrow \infty} (a_n - b_n) = \alpha - \beta$ $\lim_{n \rightarrow \infty} a_n b_n = \alpha \beta$ $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\alpha}{\beta} \quad (\beta \neq 0)$	N63a

LEVEL N

Topic	Formulas & Notes	Reference
<i>Limits of Sequences and Their Relationships</i>	1. For all n, when $a_n \leq b_n$, if $\lim_{n \rightarrow \infty} a_n = \alpha$ and $\lim_{n \rightarrow \infty} b_n = \beta$, then $\alpha \leq \beta$ if $\lim_{n \rightarrow \infty} a_n = \infty$, then $\lim_{n \rightarrow \infty} b_n = \infty$ 2. For all n, when $a_n \leq c_n \leq b_n$, if $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = \alpha$, then $\lim_{n \rightarrow \infty} c_n = \alpha$	N67a
Infinite Geometric Sequences <i>Limit of an Infinite Geometric Sequence $\{r^n\}$</i>	When $r > 1$, $\lim_{n \rightarrow \infty} r^n = \infty$... Diverges When $r = 1$, $\lim_{n \rightarrow \infty} r^n = 1$ When $r < 1$, $\lim_{n \rightarrow \infty} r^n = 0$ } ... Converges When $r \leq -1$, Oscillates (no limit) ... Diverges	N71b
Infinite Geometric Series	Given an infinite sequence $\{a_n\}$, the expression $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ ① is called the infinite series, where a_1 and a_n are called the 1st term and the n^{th} term respectively. Let S_n be the sum of the first n terms. When the infinite sequence $\{S_n\}$ converges, the infinite series ① is said to converge. When the infinite sequence $\{S_n\}$ diverges, the infinite series ① is said to diverge. Likewise, $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$ which is the infinite series that is derived from the infinite geometric sequence with 1st term a and common ratio r is called the infinite geometric series with 1st term a and common ratio r.	N81a
<i>Convergence and Divergence of an Infinite Geometric Series</i>	Given an infinite geometric series $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$, the following is true. When $a \neq 0$, if $r < 1$, then the series converges and the sum is $\frac{a}{1-r}$; if $r \geq 1$, then the series diverges. When $a = 0$, the series converges and the sum is 0.	N81b
Infinite Series	Given the infinite series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ ①, the sum of the first n terms $S_n = a_1 + a_2 + a_3 + \cdots + a_n$ is called the partial sum of the first n terms of the infinite series. The infinite series ① can be written as $\sum_{n=1}^{\infty} a_n$.	N91a
<i>Properties of Infinite Series</i>	When the infinite series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge, where $\sum_{n=1}^{\infty} a_n = S$ and $\sum_{n=1}^{\infty} b_n = T$, the following properties are true: $\sum_{n=1}^{\infty} k a_n = kS \quad (k \text{ is a constant})$ $\sum_{n=1}^{\infty} (a_n + b_n) = S + T$ $\sum_{n=1}^{\infty} (a_n - b_n) = S - T$	N95a

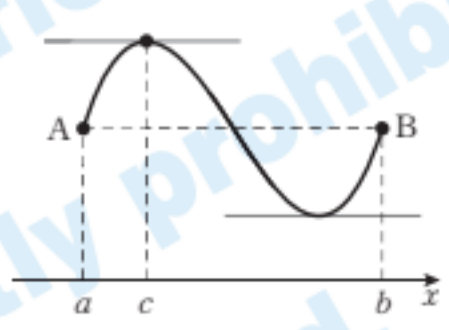
LEVEL N

Topic	Formulas & Notes	Reference
<i>Convergence and Divergence of Infinite Series</i>	If the infinite series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$. If the sequence $\{a_n\}$ does not converge to 0, then the infinite series $\sum_{n=1}^{\infty} a_n$ diverges.	N96a
Limits of Functions 1 <i>Properties of Limits and Functions</i>	If $\lim_{x \rightarrow a} f(x) = \alpha$ and $\lim_{x \rightarrow a} g(x) = \beta$, then $\lim_{x \rightarrow a} k f(x) = k \alpha \quad (k \text{ is a constant})$ $\lim_{x \rightarrow a} [f(x) + g(x)] = \alpha + \beta, \quad \lim_{x \rightarrow a} [f(x) - g(x)] = \alpha - \beta$ $\lim_{x \rightarrow a} f(x) g(x) = \alpha \beta$ $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\alpha}{\beta} \quad (\beta \neq 0)$	N101a
<i>Existence of a Limit</i>	If $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = \alpha$, then $\lim_{x \rightarrow a} f(x) = \alpha$ exists. If $\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x)$, then $\lim_{x \rightarrow a} f(x)$ does not exist.	N105a
	Given $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \alpha$, when $\lim_{x \rightarrow a} g(x) = 0$, then $\lim_{x \rightarrow a} f(x) = 0$.	N108b
Limits of Trigonometric Functions <i>Limit of $\frac{\sin x}{x}$</i>	$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	N121b
Continuous and Discontinuous Functions	Generally, the function $f(x)$ is said to be continuous at $x = a$ if $f(x)$ satisfies the following two conditions with respect to a, which is the value of x within the domain. (i) $\lim_{x \rightarrow a} f(x)$ exists. (ii) $\lim_{x \rightarrow a} f(x) = f(a)$ is true. 	N131a
<i>Intermediate Value Theorem</i>	The interval $a \leq x \leq b$ is called a closed interval and the interval $a < x < b$ is called an open interval. They are expressed as $[a, b]$ and (a, b), respectively. Generally, the Intermediate Value Theorem is explained as follows: If the function $f(x)$ is continuous on the closed interval $[a, b]$ and $f(a) \neq f(b)$, then there is at least one value c that satisfies $f(c) = k$ and $a < c < b$ for any arbitrary value k which lies between $f(a)$ and $f(b)$. 	N137a
Differentiation 1	Given the function $f(x)$, if the limit value $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ exists, then it is called the derivative value of $f(x)$ at $x = a$ and expressed as $f'(a)$. In this case, $f(x)$ is said to be differentiable at $x = a$.	N141a

LEVEL N

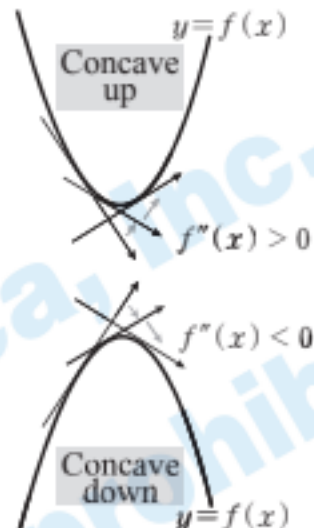
Topic	Formulas & Notes	Reference																								
Properties of Derivatives	When k is a constant and n is a positive integer, if $y = x^n$, then $y' = nx^{n-1}$ if $y = kf(x)$, then $y' = kf'(x)$ if $y = f(x) + g(x)$, then $y' = f'(x) + g'(x)$ if $y = f(x) - g(x)$, then $y' = f'(x) - g'(x)$	N143a																								
Product Rule	$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$	N144a																								
Quotient Rule	$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}, \quad \left[\frac{1}{g(x)}\right]' = -\frac{g'(x)}{[g(x)]^2}$	N146b																								
Derivative of x^n	When n is an integer, $(x^n)' = nx^{n-1}$	N148a																								
Differentiation 2 Chain Rule I	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	N151b																								
Chain Rule II	$[f(g(x))]' = f'(g(x))g'(x)$	N152a																								
Differentiation Formula for Inverse Functions	$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \quad \left(\frac{dx}{dy} \neq 0\right)$	N155a																								
Derivative of x^p	When p is a rational number, $(x^p)' = px^{p-1}$	N156a																								
Differentiation of Trigonometric Functions Derivatives of Trigonometric Functions	$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x, \quad (\tan x)' = \frac{1}{\cos^2 x}$	N162a																								
Differentiation of Logarithmic and Exponential Functions	When examining the value of $(1+k)^{\frac{1}{k}}$ by substituting k with a value which is close to 0, it approaches a constant value as shown below. e is an irrational number and $e = 2.7182818\dots$ <table><tr><th>k</th><th>$(1+k)^{\frac{1}{k}}$</th><th>k</th><th>$(1+k)^{\frac{1}{k}}$</th></tr><tr><td>0.1</td><td>2.59374...</td><td>-0.1</td><td>2.86797...</td></tr><tr><td>0.01</td><td>2.70481...</td><td>-0.01</td><td>2.73199...</td></tr><tr><td>0.001</td><td>2.71692...</td><td>-0.001</td><td>2.71964...</td></tr><tr><td>0.0001</td><td>2.71814...</td><td>-0.0001</td><td>2.71841...</td></tr><tr><td>0.00001</td><td>2.71826...</td><td>-0.00001</td><td>2.71829...</td></tr></table>	k	$(1+k)^{\frac{1}{k}}$	k	$(1+k)^{\frac{1}{k}}$	0.1	2.59374...	-0.1	2.86797...	0.01	2.70481...	-0.01	2.73199...	0.001	2.71692...	-0.001	2.71964...	0.0001	2.71814...	-0.0001	2.71841...	0.00001	2.71826...	-0.00001	2.71829...	N171a
k	$(1+k)^{\frac{1}{k}}$	k	$(1+k)^{\frac{1}{k}}$																							
0.1	2.59374...	-0.1	2.86797...																							
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0.00001	2.71826...	-0.00001	2.71829...																							
Derivatives of Logarithmic Functions I	$(\ln x)' = \frac{1}{x}, \quad (\log_a x)' = \frac{1}{x \ln a}$	N171b																								
Derivatives of Logarithmic Functions II	$(\ln x)' = \frac{1}{x}, \quad (\log_a x)' = \frac{1}{x \ln a}$	N172b																								

LEVEL N

Topic	Formulas & Notes	Reference
<i>Derivatives of Exponential Functions</i>	$(e^x)' = e^x, \quad (a^x)' = a^x \ln a$	N177a
Differentiation of Various Functions and Higher Order Derivatives <i>Derivative of Functions Represented by a Parameter</i>	When $x = f(t)$ and $y = g(t)$, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$	N183a
	In general, the function obtained by differentiating the function $y = f(x)$ n times is called the n^{th} order derivative of $f(x)$ and can be written as $y^{(n)}, f^{(n)}(x), \frac{d^n y}{dx^n}, \text{ or } \frac{d^n}{dx^n} f(x)$	N185a N189a
Various Properties of Derivatives <i>Rolle's Theorem</i>	If the function $f(x)$ is continuous on the closed interval $[a, b]$, differentiable on the open interval (a, b) and $f(a) = f(b)$, then there exists at least one value c such that $f'(c) = 0$ and $a < c < b$. 	N196a
<i>Mean Value Theorem</i>	If the function $f(x)$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , then there exists at least one value c such that $\frac{f(b) - f(a)}{b - a} = f'(c)$ and $a < c < b$.	N197a

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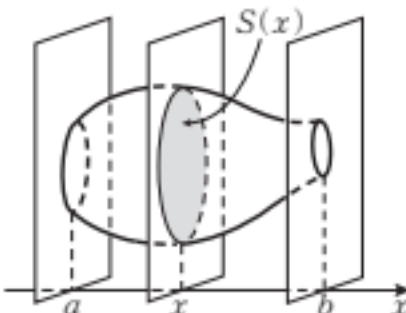
LEVEL O

Topic	Formulas & Notes	Reference
Tangents and Normals <i>Equation of a Tangent Line</i>	The equation of the tangent to the curve $y=f(x)$ at point $(a, f(a))$ is $y-f(a)=f'(a)(x-a)$	O1a
<i>Equation of a Normal Line</i>	The equation of the normal to the curve $y=f(x)$ at point $(a, f(a))$ is $y-f(a)=-\frac{1}{f'(a)}(x-a) \text{ when } f'(a) \neq 0$ $x=a \text{ when } f'(a)=0$	O3a
Concavity of Curves <i>Concave Up</i> <i>Concave Down</i>	When a function $f(x)$ has the second order derivative $f''(x)$, in the interval where $f''(x) > 0$, the curve $y=f(x)$ is concave up; and in the interval where $f''(x) < 0$, the curve $y=f(x)$ is concave down. 	O21a
Various Applications of Differentiation <i>Linear Approximation I</i>	When the function $f(x)$ is differentiable at $x=a$ and the value of $ h $ is approaching 0, $f(a+h) \approx f(a)+f'(a)h$	O48a
<i>Linear Approximation II</i>	When the value of $ x $ is approaching 0, $f(x) \approx f(0)+f'(0)x$	O48b
Indefinite Integrals 1	The function which becomes $f(x)$ when differentiated is called the indefinite integral or antiderivative of $f(x)$ and is expressed as $\int f(x)dx$. Let an indefinite integral of $f(x)$ be $F(x)$. Then, it is expressed as $\int f(x)dx=F(x)+C \quad (C \text{ is the constant of integration})$	O51a
<i>Indefinite Integral of x^a</i>	$\int x^a dx=\frac{1}{a+1}x^{a+1}+C \quad (a \neq -1)$ $\int x^{-1} dx=\int \frac{1}{x} dx=\ln x +C$	O51a
<i>Properties of Indefinite Integrals</i>	$\int kf(x)dx=k \int f(x)dx \quad (k \text{ is a constant})$ $\int [f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$ $\int [f(x)-g(x)]dx=\int f(x)dx-\int g(x)dx$	O52a
<i>Indefinite Integral of $(ax+b)$</i>	When $F'(x)=f(x)$, $a \neq 0$, $\int f(ax+b)dx=\frac{1}{a}F(ax+b)+C$	O53a

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Topic	Formulas & Notes	Reference				
Indefinite Integrals 2 <i>Indefinite Integrals of Exponential Functions</i>	$\int e^x dx = e^x + C, \quad \int a^x dx = \frac{a^x}{\ln a} + C$	O61a				
<i>Indefinite Integrals of Trigonometric Functions</i>	$\int \sin x \, dx = -\cos x + C, \quad \int \cos x \, dx = \sin x + C$ $\int \frac{1}{\cos^2 x} dx = \tan x + C, \quad \int \frac{1}{\sin^2 x} dx = -\frac{1}{\tan x} + C$	O62a				
Integration by Substitution <i>Integration by Substitution I</i>	$\int f(x) dx = \int f(g(t)) g'(t) dt \quad (x = g(t))$	O71a				
<i>Integration by Substitution II</i>	$\int f(g(x)) g'(x) dx = \int f(u) du \quad (g(x) = u)$	O74a				
<i>Indefinite Integral of $\frac{g'(x)}{g(x)}$</i>	$\int \frac{g'(x)}{g(x)} dx = \ln g(x) + C$	O78a				
Integration by Parts	$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$	O81a				
Definite Integrals	If $F(x)$ is an indefinite integral (antiderivative) of $f(x)$, then $\int_a^b f(x) dx = \left[F(x) \right]_a^b = F(b) - F(a)$	O91a				
<i>Property of Definite Integrals I</i>	$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$	O97a				
Integration by Substitution for Definite Integrals	When $x = g(t)$, if $a = g(\alpha)$ and $b = g(\beta)$, then $\int_a^b f(x) dx = \int_\alpha^\beta f(g(t)) g'(t) dt$ <table style="display: inline-table; vertical-align: middle;"><tr><td style="border-right: 1px solid black; padding: 0 5px;">x</td><td style="padding: 0 5px;">$a \longrightarrow b$</td></tr><tr><td style="border-right: 1px solid black; padding: 0 5px;">t</td><td style="padding: 0 5px;">$\alpha \longrightarrow \beta$</td></tr></table>	x	$a \longrightarrow b$	t	$\alpha \longrightarrow \beta$	O101a
x	$a \longrightarrow b$					
t	$\alpha \longrightarrow \beta$					
<i>Property of Definite Integrals II</i>	$\int_b^a f(x) dx = -\int_a^b f(x) dx$	O102a				
<i>Integration of Even/Odd Functions</i>	When $f(x)$ is an even function, $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ When $f(x)$ is an odd function, $\int_{-a}^a f(x) dx = 0$	O107b				
Integration by Parts for Definite Integrals and Functions Represented by Definite Integrals <i>Integration by Parts for Definite Integrals</i>	$\int_a^b f(x) g'(x) dx = \left[f(x) g(x) \right]_a^b - \int_a^b f'(x) g(x) dx$	O111a				

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Topic	Formulas & Notes	Reference
<i>Definite Integrals and Differentiation</i>	When a is a constant, $\frac{d}{dx} \int_a^x f(t) dt = f(x)$	O115a
<i>Property of Definite Integrals III</i>	$\int_a^a f(x) dx = 0$	O116b
Integration by Quadrature and Proof of Inequalities <i>Definite Integrals and Limits of Sums I</i>	If the function $f(x)$ is continuous on the interval $[a, b]$, $\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} f(x_k) \Delta x = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x = \int_a^b f(x) dx$ where $\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x$	O122a
<i>Definite Integrals and Limits of Sums II</i>	$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} f\left(\frac{k}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$	O122b
Areas <i>Curve $y = f(x)$ and Area</i>	On the interval $[a, b]$, when $f(x) \geq 0$, $S = \int_a^b f(x) dx$ when $f(x) \leq 0$, $S = -\int_a^b f(x) dx$	O131a
<i>Area between Two Curves</i>	On the interval $[a, b]$, when $f(x) \geq g(x)$, $S = \int_a^b [f(x) - g(x)] dx$	O132a
<i>Curve $x = g(y)$ and Area</i>	On the interval $c \leq y \leq d$, when $g(y) \geq 0$, $S = \int_c^d g(y) dy$	O134a
Volumes	Let V be the volume of the solid between the planes $x=a$ and $x=b$, where $a < b$. Letting $S(x)$ be the area of the intersection of the plane at x with the solid, $V = \int_a^b S(x) dx$ 	O141b
<i>Volume of Revolution about the x-axis</i>	$V = \pi \int_a^b [f(x)]^2 dx = \pi \int_a^b y^2 dx$	O143a
<i>Volume of Revolution about the y-axis</i>	$V = \pi \int_c^d [g(y)]^2 dy = \pi \int_c^d x^2 dy$	O144a
Length of a Curve Velocity and Distance <i>Length of a Curve I</i>	The length L of the curve $x=f(t)$, $y=g(t)$ ($a \leq t \leq b$) is $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2} dt$	O151b

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Topic	Formulas & Notes	Reference
<i>Length of a Curve II</i>	The length L of the curve $y = f(x)$ ($a \leq x \leq b$) is $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b \sqrt{1 + [f'(x)]^2} dx$	O153a
<i>Displacement and Distance of a Point Moving on a Line</i>	$s = \int_a^b v dt, \quad l = \int_a^b v dt$	O155a
<i>Change in the Velocity of a Point Moving on a Line</i>	Let the velocity of point P moving on a number line at time t_0 and t_1 be v_0 and v_1 respectively, and let the acceleration at time t be α. $v_1 = v_0 + \int_{t_0}^{t_1} \alpha dt$	O156a
<i>Distance of a Point Moving on a Plane</i>	$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$	O157a
Differential Equations	The differential equation expressed as $f(y) \frac{dy}{dx} = g(x)$ is called a separable differential equation. $\frac{dy}{dx} = \frac{x}{y}$ [Sol] $y \frac{dy}{dx} = x \quad \leftarrow$ Rearranging into the form $f(y) \frac{dy}{dx} = g(x)$ Integrating both sides with respect to x, $\int y dy = \int x dx \quad \leftarrow \quad \text{LHS} = \int y \frac{dy}{dx} \cdot dx = \int y dy$ $\therefore \frac{1}{2} y^2 = \frac{1}{2} x^2 + C_1$ $\therefore y^2 = x^2 + 2C_1$ Let $2C_1 = C$. $y^2 = x^2 + C \quad (C \text{ is an arbitrary constant}) \quad \leftarrow$ Generally, arbitrary constants do not have coefficients.	O164a

[NOTES]